hep-th/9510074, NYU-TH-95/10/02

CERN-TH/95-268, UCLA/95/TEP/33

Minimal Higgs Branch for the Breaking of Half of the Supersymmetries in N=2 Supergravity

Sergio Ferrara[a], Luciano Girardello[b] and Massimo Porrati[c],

a Theory Division, CERN, CH-1211 Geneva 23, Switzerland

b Theory Division, CERN, CH-1211 Geneva 23, Switzerland [1]

c Department of Physics, NYU, 4 Washington Pl., New York NY 10003, USA [2]

ABSTRACT

It is shown that the minimal Higgs sector of a generic N=2 supergravity theory with unbroken N=1 supersymmetry must contain a Higgs hypermultiplet and a vector multiplet. When the multiplets parametrize the quaternionic manifold $SO(4,1)/SO(4)$, and the special Kähler manifold $SU(1,1)/U(1)$, respectively, a vanishing vacuum energy with a sliding massive spin 3/2 multiplet is obtained. Potential applications to N=2 low energy effective actions of superstrings are briefly discussed.

[1]On leave from Dipartimento di Fisica, Università di Milano, via Celoria 16, 20133 Milano, Italy.

[2]On leave from INFN, Sez. di Pisa, 56100 Pisa, Italy.

The field theoretical analysis of the Higgs and the super-Higgs mechanisms has already proven itself in the past to be a powerful tool to analyse phenomena that may occur in string theory.

Recently, conifold transitions in type II strings compactified on Calabi-Yau manifolds [1] have been described by Greene, Morrison and Strominger [2] as a Higgs mechanism in which p hypermultiplets are "eaten up" by p $U(1)$ massless vector multiplets, which become p massive, long vector multiplets [3] (i.e. multiplets with vanishing central charge), each with spin content $(1, 1/2(4), 0(5))$. This Higgs branch is a particular case of a phenomenon, previously noted in the context of supersymmetric gauge theories [4], where, generically, VEV's of hypermultiplets can change the rank of the (unbroken) gauge group. This is is contrast with the Coulomb phase in which VEV's of vector multiplets do not change the rank of the gauge group.

Purpose of the present work is to investigate a much richer structure which emerges in supergravity theories, in which these branches can induce supersymmetry breaking together with gauge symmetry breaking.

The new phenomenon which emerges here is that N=2 Fayet-Iliopoulos terms [5] can break all or half [6] of the supersymmetries depending on whether charged hypermultiplets exist in the theory [6], which couple both to the graviphoton and to the matter vector multiplets. Partial supersymmetry breaking is also possible with vanishing vacuum energy [7].

Suppose at first that hypermultiplets are not present, but that Fayet Iliopoulos terms are. Furthermore, choose an Abelian gauge group $U(1)^{n_V+1}$, where n_V is the number of matter vector hypermultiplets. Then the Fayet-Iliopoulos term corresponds to a constant gauge prepotential [8]

$$\mathcal{P}^x_\Lambda = \xi^x_\Lambda, \tag{1}$$

such that:

$$(\vec{\xi}_\Lambda \wedge \vec{\xi}_\Sigma)^z = \epsilon^{xyz}\mathcal{P}^x_\Lambda \mathcal{P}^y_\Sigma = 0 \tag{2}$$

The vacuum energy, in the notations of reference [8], is given by the formula

$$V(z, \bar{z}, \xi^x_\Lambda) = U^{\Lambda\Sigma}\xi^x_\Lambda \xi^x_\Sigma - 3\bar{L}\cdot\xi^x L\cdot\xi^x. \tag{3}$$

Here z denotes the scalars of the vector-multiplet manifold, whose metric is $g_{i\bar{\jmath}}$, while

$$U^{\Lambda\Sigma} = (\partial_i + \frac{1}{2}\partial_i K)L^\Lambda g^{i\bar{\jmath}}(\partial_{\bar{\jmath}} + \frac{1}{2}\partial_{\bar{\jmath}} K)\bar{L}^\Sigma, \quad L^\Lambda = e^{K/2}X^\Lambda. \tag{4}$$

The special geometry data X^Λ, K are defined below, in the parargraph after eq. (10). In reference [5] it was shown that for a particular choice of the prepotential, $F(X^\Lambda)$, it is possible to have $V \equiv 0$. However, since $\langle z\rangle$ is $SU(2)$ invariant, it follows that both gravitini have the same (sliding) mass [6]. Therefore, this example breaks all supersymmetries, while the $U(1)^{N_V+1}$ symmetry is unbroken. It thus corresponds to the Coulomb phase. In this case gauginos get masses proportional to the gravitino mass.

A more interesting situation arises when the theory is not in the Coulomb phase, but rather in the Higgs phase. As in rigid supersymmetry, this can only occur if matter hypermultiplets are present. The new phenomenon that we want to emphasize here is that the hypermultiplets

not only can give masses to the $U(1)^{n_V+1}$ gauge bosons, but can also break half of the supersymmetries rather than all of them, in the presence of Fayet-Iliopoulos terms [7]. Interestingly enough this is a phenomenon that has no analog in rigid supersymmetric theories (whether or not they are renormalizable) simply because, as pointed out by Witten [9], in rigid theories the supersymmetry algebra implies that if one supersymmetry is broken, then the vacuum energy is strictly positive, implying that *all* supersymmetries are indeed broken. In supergravity this is circumvented because the supergravity Ward identities read [10]

$$\delta_A \psi_L^i \delta^B \psi_R^j \mathcal{Z}_{ij} - 3\mathcal{M}_{AC}\bar{\mathcal{M}}^{CB} = V\delta_A^B, \tag{5}$$

Where the $\delta_A \psi^i$ denote the shift, under the A-th supersymmetry, of the spin one-half fermions, while $\mathcal{M}_{AB}$ is the gravitino mass matrix and $\mathcal{Z}_{ij}$ is the kinetic term of the fermions. These identities show that even when $V = 0$, one may still have, say

$$\delta_1 \psi_L^i \delta^1 \psi_R^j \mathcal{Z}_{ij} = 3\mathcal{M}_{1C}\mathcal{M}^{C1} = 0, \tag{6}$$

but instead

$$\delta_2 \psi_L^i \delta^2 \psi_R^j \mathcal{Z}_{ij} = 3\mathcal{M}_{2C}\mathcal{M}^{C2} \neq 0. \tag{7}$$

In N=2, this corresponds to breaking half of the supersymmetries (N=1 unbroken), at zero cosmological constant.

A model that realizes such a situation cannot be obtained from the Lagrangian of De Wit, Lauwers and Van Proeyen [11], as it was proven in [12]. On the other hand, that is not the most general N=2 Lagrangian. It uses, in fact, a symplectic basis in which a prepotential $F(X)$ exists for the vector multiplets. In reference [13], it was shown that this is not generally true, and that a more general formulation of N=2 supergravity exists, that never makes use of the prepotential function.

The minimal model that exhibit partial breaking of $N = 2$ supersymmetry to $N = 1$ with zero cosmological constant contains a charged hypermultiplet, whose scalars parametrize the quaternionic manifold $SO(4,1)/SO(4)$, coupled to a vector multiplet, whose scalars parametrize the Kähler manifold $SU(1,1)/U(1)$ [3]. The latter is formulated in a symplectic basis in which no prepotential exists.

Note that the presence of both a hypermultiplet and a vector multiplet is needed [14] since, when N=2 is broken to N=1, the N=1 multiplet containing the massive spin-3/2 field has spin content $(3/2, 1, 1, 1/2)$. Both the graviphoton and the matter vector become massive, together with one of the gravitini; in other words, this is a Higgs and super-Higgs phase. The spectrum of this theory contains, besides the massive spin-3/2 N=1 multiplet, two massless chiral multiplets with sliding fields, since the vacuum energy vanishes.

The model is determined by the geometry of the hypermultiplet quaternionic manifold and the geometry of the vector-multiplet manifold, together with the the "D-term" prepotentials $\mathcal{P}_\Lambda^x$ [8].

[3] This model was constructed in [7] by performing a singular limit on a model constructed within the framework of the tensor calculus of ref. [11].

Let us denote the quaternionic coordinates of the hypermultiplet manifold by b^u, $u = 0,1,2,3$. The quaternionic geometry is determined by a triplet of quaternionic potentials, $\Omega^x = \Omega^x_{uv} db^u \wedge db^v$, $x = 1,2,3$, which are the field strenght of an $SU(2)$ connection $\omega^x = \omega^x_u db^u$: $\Omega^x = d\omega^x + (1/2)\epsilon^{xyz}\omega^y \wedge \omega^z$. In our case, the quaternionic manifold is $SO(4,1)/SO(4)$, and these quantities read:

$$\omega^x_u = \frac{1}{b^0}\delta^x_u, \quad \Omega^x_{0u} = -\frac{1}{2b^{0\,2}}\delta^x_u, \quad \Omega^x_{yz} = \frac{1}{2b^{0\,2}}\epsilon^{xyz}, \quad x,y,z = 1,2,3. \tag{8}$$

The prepotentials Ω^x determine the quaternionic metric h_{uv} by the identity [8]

$$h^{st}\Omega^x_{us}\Omega^y_{tv} = -\delta^{xy}h_{uv} - \epsilon^{xyz}\Omega^z_{uv}. \tag{9}$$

In our case this equation gives $h_{uv} = (1/2b^{0\,2})\delta_{uv}$. To write the fermion shifts one also needs the symplectic vielbein $\mathcal{U}^{\alpha A}_u db^u$, $\alpha, A = 1,2$ [8]. In our case the vielbein reads:

$$\mathcal{U}^{\alpha A} = \frac{1}{2b^0}\epsilon^{\alpha\beta}(db^0 - i\sigma^x db^x)_\beta^{\ A}, \tag{10}$$

where σ^x are the standard Pauli matrices.

The special geometry of the manifold of the vector multiplets is determined in general by giving $2n_V + 2$ holomorphic sections [13] $X^\Lambda(z), F_\Lambda(z)$, in terms of which the Kähler potential reads

$$K = -\log i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda). \tag{11}$$

Here the manifold is $SU(1,1)/U(1)$, $\Lambda = 0,1$, there is a single holomorphic coordinate z, and our choice of holomorphic sections is

$$X^0(z) = -\frac{1}{2}, \quad X^1(z) = \frac{i}{2}, \quad F_0 = iz, \quad F_1 = z. \tag{12}$$

This choice gives rise to the Kähler potential

$$K = -\log(z + \bar{z}), \tag{13}$$

and thus to the metric $g_{z\bar{z}} = 1/(z+\bar{z})^2$. It is important to remark that our choice of holomorphic sections is such that no prepotential $F(X^\Lambda)$ exists [13] [4].

Any global symmetry of the hypermultiplet manifold can be gauged. If the corresponding Killing vectors are k^u_Λ, the gauge covariant derivative is $D_\mu b^u = \partial_\mu b^u + A^\Lambda_\mu k^u_\Lambda$. In our case the gauge group is $U(1)^2$, where one of the $U(1)$ factors comes from the N=2 graviphoton, and the other from the matter vector. Therefore, we need two commuting Killing vectors. Since the metric of our quaternionic manifold is $\delta_{uv}(1/2b^{0\,2})$, the manifold is symmetric under arbitrary constant translation of the coordinates b^1, b^2, b^3. Thus, we can for instance choose to gauge the translations along b^1 with the graviphoton, and the translations along b^2 with the matter vector. The corresponding Killing vectors are

$$k^u_0 = g\delta^{u1}, \quad k^u_1 = g'\delta^{u2}, \tag{14}$$

[4] One can find these sections by the symplectic transformation $X^1 \to -F_1$, $F_1 \to X^1$ of the basis specified by the prepotential $F(X^\Lambda) = iX^0X^1$, which reads, explicitly, X^Λ,$F_\Lambda = \partial F/\partial X^\Lambda$.

where g and g' are arbitrary constants (the gauge couplings of the two $U(1)$'s). The Killing vectors of a quaternionic manifold are derived from a triplet of "D-term" prepotentials, $\mathcal{P}^x_\Lambda$ by the equation [8]

$$k^u_\Lambda = \frac{1}{6}\sum_{x=1}^{3} h^{vw}\nabla_v \mathcal{P}^x \Omega^x_{wt} h^{tu}. \tag{15}$$

In our case one has

$$\mathcal{P}^x_0 = g\frac{1}{b^0}\delta^{x1}, \quad \mathcal{P}^x_1 = g'\frac{1}{b^0}\delta^{x2}. \tag{16}$$

It is easily checked that the "quaternionic Poisson braket [8]" of these prepotentials is zero, as it must be for an Abelian gauge group

$$\{\mathcal{P}_0, \mathcal{P}_1\}^x \equiv \Omega^x_{uv} k^u_0 k^v_1 - \frac{1}{2}\epsilon^{xyz}\mathcal{P}^y_0 \mathcal{P}^z_1 = 0. \tag{17}$$

The relation between our prepotentials and Killing vectors can be summarized by the following formula

$$\mathcal{P}^x_\Lambda = \frac{1}{b^0} k^x_\Lambda. \tag{18}$$

At this point, we have determined all quantities necessary to write the fermion shifts. The formulae of reference [8] give the following expression for the (constant part of the) gaugino shift

$$\delta\lambda^{\bar{z}}_A = -ig^{z\bar{z}}(\sigma^x)_A^{\ C}\epsilon_{BC}\mathcal{P}^x_\Lambda e^{K/2}(\partial_z + \partial_z K)X^\Lambda(z)\eta^B \equiv W^{\bar{z}}_{AB}\eta^B. \tag{19}$$

Here η^A is the N=2 supersymmetry parameter.

The shifts of the hyperini is instead

$$\delta\zeta^\alpha = -2\epsilon_{AB}\mathcal{U}^{\alpha B}_u k^u_\Lambda e^{K/2} X^\Lambda(z)\eta^A \equiv \mathcal{N}^\alpha_A \eta^A. \tag{20}$$

Finally the gravitino shift reads

$$\delta\psi_{A\,\mu} = \frac{i}{2}(\sigma^x)_A^{\ C}\epsilon_{BC}\mathcal{P}^x_\Lambda e^{K/2} X^\Lambda(z)\gamma_\mu \eta^B \equiv iS_{AB}\gamma_\mu\eta^B. \tag{21}$$

By substituting into these formulas the explicit expressions we obtained for all quantities involved, we find that all fermionic shifts are proportional to a single matrix:

$$W^{\bar{z}}_{AB} = -i(z+\bar{z})^{1/2}\frac{1}{b^0}X_{AB}, \;\; \mathcal{N}^\alpha_A = -i(z+\bar{z})^{-1/2}\frac{1}{b^0}\epsilon^{\alpha\beta}X_{\beta A}, \;\; S_{AB} = -\frac{1}{2}(z+\bar{z})^{-1/2}\frac{1}{b^0}X_{AB}, \tag{22}$$

where

$$X_{AB} = -\frac{g}{2}(\sigma^1)_A^{\ C}\epsilon_{CB} + i\frac{g'}{2}(\sigma^2)_A^{\ C}\epsilon_{CB} = \begin{pmatrix} \frac{g'-g}{2} & 0 \\ 0 & \frac{g'+g}{2} \end{pmatrix}. \tag{23}$$

In these normalizations, the Ward identity relating the scalar potential to the fermionic shifts reads

$$\delta_{AB}V = -12(S_{AC})^* S_{CB} + g_{z\bar{z}}(W^{\bar{z}}_{AC})^* W^{\bar{z}}_{CB} + 2(\mathcal{N}^\alpha_A)^*\mathcal{N}^\alpha_B. \tag{24}$$

Upon substituting eq. (23), we find that this formula gives $V = 0$ identically, for *any* value of g and g'. The model has always a flat potential, and sliding VEVs for the scalar fields z and b^u.

The gravitino mass matrix is equal to $2S_{AB}$ (compare eq. (5) with eq. (24)); thus, the ratio of the mass of the two gravitini is independent on the scalar VEV and equal to $|(g+g')/(g-g')|$. When the gauge coupling of the two $U(1)$'s are equal in magnitude $g = \pm g'$ (and nonzero), one of the two gravitini is massless, and N=1 supersymmetry is unbroken. Obviously, all the fermionic shifts along the unbroken supersymmetry generator vanish.

It is apparent that the model presented here describes the minimal sector responsible for the breaking of half of the supersymmetries. It is thus conceivable that its Lagrangian would also provide a model-independent description of the interactions of the half-supersymmetry breaking sector of a very large class of interesting theories.

We may wonder whether phenomena such as have been just described here may occur in string theory. If the N=2 theory under consideration is coming from a type IIA theory, then the vectors are R-R states, and the hypermultiplets carry R-R charges [2, 15]. On the other hand, the breaking of half of the supersymmetries is only possible if a Fayet-Iliopoulos term is introduced. In our case the prepotentials $\mathcal{P}^x_\Lambda$ are Fayet-Iliopoulos terms since, as shown by eq. (16), they are independent of the vector multiplets and they are always nonzero at any point on the hypermultiplet manifold.

It is interesting to remark that, as recently noted in ref. [16], a kind of Fayet-Iliopoulos term was introduced by Romans [17] in type IIA supergravity. It induces an anti-Higgs mechanism for a $U(1)$ 10-D vector field, which is eaten by the $b_{\mu\nu}$ tensor, that thus becomes massive. Ref. [16] also discusses a 10-D form in type II theory which induces a supersymmetry breaking in string theory. It is plausible that the mechanism discussed in this paper, or a generalization thereof, may find applications in the understanding of non-perturbative phenomena in superstring dynamics.

Acknowledgements

S.F. is supported in part by DOE under grant DE-FGO3-91ER40662, Task C, and by EEC Science Program SC1*-CI92-0789. L.G. is supported in part by Ministero dell' Università e della Ricerca Scientifica e Tecnologica, by INFN, and by EEC Science Programs SC1*-CI92-0789 and CHRX-CT92-0035. M.P. is supported in part by NSF under grant PHY-9318781.

References

[1] A. Strominger, Nucl. Phys. B451 (1995) 96.

[2] B.R. Greene, D.R. Morrison and A. Strominger, Nucl. Phys. B451 (1995) 109.

[3] S. Ferrara, C.A. Savoy and B. Zumino, Phys. Lett. 100B (1981) 393.

[4] N. Seiberg and E. Witten, Nucl. Phys. B431 (1994) 484.

[5] B. de Wit, P.G. Lauwers, R. Philippe, S.Q. Su and A. Van Proeyen, Phys. Lett. 134B (1984) 37; B. de Wit and A. Van Proeyen, Nucl. Phys. B245 (1984) 89; E. Cremmer, C.

Kounnas, A. Van Proeyen, J.P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, Nucl. Phys. B250 (1985) 385.

[6] S. Cecotti, L. Girardello and M. Porrati, Phys. Lett. 151B (1985) 367.

[7] S. Cecotti, L. Girardello and M. Porrati, Phys. Lett. 168B (1986) 83.

[8] R. D' Auria, S. Ferrara and P. Fré, Nucl. Phys. B359 (1991) 705.

[9] E. Witten, Nucl. Phys. B188 (1981) 513.

[10] S. Cecotti, L. Girardello and M. Porrati, Nucl. Phys. B268 (1986) 265; *Proceedings of the IX Johns Hopkins Workshop* (World Scientific, Singapore, 1985); S. Ferrara and L. Maiani, *Proceedings of the V SILARG Symposium* (World Scientific, Singapore, 1986).

[11] B. de Wit, P.G. Lauwers and A. Van Proeyen, Nucl. Phys. B255 (1985) 569.

[12] S. Cecotti, L. Girardello and M. Porrati, Phys. Lett. 145B (1984) 61.

[13] A. Ceresole, R. D' Auria, S. Ferrara and A. Van Proeyen, Nucl. Phys. B444 (1995) 92.

[14] S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. 127B (1983) 70.

[15] *For a recent review, see:* A. Ceresole, R. D' Auria and S. Ferrara, preprint CERN-TH/95/244, hep-th/9509160.

[16] J. Polchinski, preprint NSF-ITP-45-122, hep-th/9510017.

[17] L.J. Romans, Phys. Lett. 169B (1986) 374.